UNIFORM CIRCULAR MOTION, CENTRIPETAL FORCE

When an object travels at a constant speed (equal distances in equal time intervals) along the circumference of a circle, the motion is called uniform circular motion. The diagram below shows the path of the moving object along with the velocity vector $\mathbf{v}$ and the acceleration vector $\mathbf{a}$. Although the speed of the object (defined as the magnitude of the velocity) is a constant, the velocity vector itself is not, as it constantly changes its direction. The diagram shows the position and velocity of the object at two instants of time separated by a time interval $\Delta t$. Note that the two velocity vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same magnitude but differ in direction by the same angle as the two radii to the two points. In fact, if one draws the two vectors from the same point (as shown) one can determine the vector difference $\Delta \mathbf{v}$ as shown. $\Delta \mathbf{v}$ is defined as $\mathbf{v}_2 - \mathbf{v}_1$ and is therefore a vector that when added to $\mathbf{v}_1$ gives $\mathbf{v}_2$. This is clearly true in the vector addition diagram. Note that the angle between $\Delta \mathbf{v}$ and $\mathbf{v}_1$ is very nearly $90^\circ$. Imagine two points on the circle that are extremely close to each other. Using two such points, the vector $\Delta \mathbf{v}$ would become extremely close to the radius direction (perpendicular to the velocity vector). The acceleration $\mathbf{a}$ is defined to be $\Delta \mathbf{v}/\Delta t$ and is therefore in the same direction as the vector $\Delta \mathbf{v}$, i.e., along the radius and toward the center.

Motion along a circle is therefore an accelerated motion. The acceleration $\mathbf{a}$ points toward the center of the circle. It is therefore called CENTRIPETAL ACCELERATION. From Newton’s laws, if an object is to have such an acceleration, a force $\mathbf{F}$ must be applied to it. The motion that results in the circle path is caused by this centripetal force. It is up to the agent producing the motion to provide this necessary force. In the motion of the Earth about the Sun (approximately circular) the force is provided by the gravitational attraction of the Sun. In the motion of a stone at the end of a string (going along a circular path) the tension in the string provides the necessary force.

The magnitude of the centripetal acceleration is given by:

$$a = \frac{v^2}{R}$$

where $v$ is the speed (magnitude of the velocity vector) and $R$ is the radius of the circle. The direction of $a$ is **toward the center of the circular path**.

---

*The lab instructions for this experiment were written by R. Warasila et al. of Suffolk County Community College.*
The Centripetal Force $F$ is given by $F = Ma$ where $m$ is the mass of the object going around the circle and $a$ is the centripetal acceleration. $F$ is therefore in the same direction as $a$ (toward the center) and its magnitude is given by

$$F_{\text{cent}} = \frac{Mv^2}{R} \text{ in Newtons}$$

In this experiment the $F_{\text{cent}}$ will be provided by a spring. You will measure the size of the force the spring provides and then set the mass into circular motion and measure the $v$. After these measurements you can compare the spring force to the centripetal acceleration.

**Procedure:**

The apparatus in the experiment consists of a mass $m$ attached to a horizontal spring (that provides the centripetal force) and hung from a string (to compensate for the force of gravity). The mass rotates about a circular path as shown.

Here are some important aspects of the apparatus to understand:

1. When the mass $m$ is in motion around a circle, the string holding it must be in a **vertical** position. This property is shown in the diagrams. Under these conditions, the weight of the mass ($Mg$) is totally compensated by the tension in the string. The only remaining force would therefore be the force of the spring that will provide the horizontal centripetal force. If the string is **not** vertical there will be an additional horizontal force from the string.

2. The extra weight attached to the bar at the top is for maintaining equilibrium. Adjust it whenever you change the radius of the motion so that the center of mass is positioned over the axis. The bar on the top of the apparatus can be moved to change the distance of the mass from the center of rotation. Adjusting the balancing weight will result in smaller frictional forces allowing the apparatus to rotate longer without an appreciable slowing down. If you observe the platform is wobbling, adjust the counter weight.
3. You will find that due to friction the radius of the motion may not remain a constant long enough for you to measure the time for 25 revolutions. You can help the motion along manually, by rotating the vertical bar, keeping the radius a constant. This technique will improve your results. The horizontal bar holding the mass should be adjusted whenever the radius is changed. One common mistake is to move the pointer first and then neglect to adjust the horizontal bar. This would imply that the vertical string holding the mass would not be truly vertical.

Set the mass into motion in a circular path by rotating the main vertical bar by hand (gently). Increase the velocity until the mass begins to rotate such that the string holding it will be exactly vertical. The quantities involved (m, v, R, and F) are measured as follows:

**Measurement of v:** To measure the speed v, simply time the motion of the mass for 25 rotations. If the period of motion is defined to be T, then the period is given by:

\[
\text{Time for 25 revolutions})/25.
\]

Furthermore the period T is simply related to v since:

\[
T = \frac{\text{Distance covered}}{\text{speed}} = \frac{2\pi R}{v}.
\]

Therefore:

\[
T = \frac{2\pi R}{v} \quad \text{or} \quad v = \frac{2\pi R}{T}
\]

Where R is the radius of the circle measured from the center of the mass to the center of rotation. The speed v can therefore be computed from the knowledge of R and T. Notice how critical the measurement of R is to this determination. Therefore be very careful that the mass moves vertically over the post.

**Measurement of the force F:** The force F is given by \( F = Mv^2/r \). It can therefore be calculated from R, v and M. to compare this calculated value of F with the measurement, one must know the force in the spring when it is stretched to the same point as when the mass m was revolving in a circle over the post. Imagine that the spring came with a dial indicating the force in the spring while it was rotating! One would then simply read the dial and that would be the measurement of F. Unfortunately the springs do not come with such dials. Therefore an alternative method of measuring the force in the spring is required.

1. Stop the rotation and connect a string to the mass. Pass the string over the pulley and connect it to a weight holder.
2. Place enough weights on the weight holder to stretch the spring to the same point as it was when the mass was rotating. The force in the spring must be the same now as it was when the mass was rotating.
3. Let the mass required to balance the spring in this part of the experiment (including the mass of the weight holder itself) be \( m_1 \). The force in the spring would then be given:

\[
\text{Force in the spring} = \text{weight of the mass } m_1 = m_1 g.
\]

**NOTE** that this is NOT the mass M that moves in the circular path!
Make measurements with 3 different radius (R) settings.

Presentation

Data and Analysis Table mixed (separate them in your report, and add more columns if you wish)

<table>
<thead>
<tr>
<th>Radius R</th>
<th>Time for 25 Rotations</th>
<th>Period T</th>
<th>Spring Force m_1g</th>
<th>Speed v = 2πR/T</th>
<th>Theoretical Force Mv^2/R</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>±</td>
</tr>
</tbody>
</table>

In your presentation or error analysis include comments on the following concepts:

1. What part of the equipment provides the centripetal force $F_{cent}$?
2. Why is the weight (Mg) of the mass M that is following the circular path ignored?
3. Why do we measure the period by timing 25 revolutions? Why not one revolution or 100 revolutions?

As usual report a % error based on the last column. Is this error consistent with the uncertainties?
Optional to the Instructor

Your instructor may require you answer these questions on a separate sheet attached to your lab report.

1. Derive the formula for centripetal acceleration \( a_{\text{cent}} = \frac{v^2}{R} \).

2. What is the direction of this acceleration?

3. A particle is moving at constant speed \( v \) in a circle of radius \( R \).
   What is meant by the period of the motion?
   What is meant by the frequency of the motion?
   How are the frequency and the period related?

4. Derive the relation between the speed of motion along a circular path \( v \) and the angular speed \( \omega \) (i.e., \( v = \omega r \)). What are the units of \( \omega \)?

5. Distinguish between the tangential acceleration \( a_{\text{tan}} \) of motion along the circumference of a circle and the centripetal acceleration \( a_{\text{cent}} \). How is each of these quantities related to the angular acceleration \( \alpha \)?

6. Derive the expression for the centripetal force \( F_{\text{cent}} = m\omega^2R \) in the terms of the angular velocity.

7. A particle is moving at constant speed in a circle of radius 2 meters. It takes 20 seconds to make 4 complete revolutions. What is the angular velocity, the period, and the linear speed \( v \)? What is its angular acceleration?

8. In the previous problem if the mass of the object is 3 kg, what was the magnitude and direction of the applied force responsible for the motion?

9. What is wrong with the following statement?
   “A mass being swung in a circle by a string causes a centripetal force to be exerted on the string.”

10. How could you measure an unknown mass \( m \) in a spaceship far in space (no gravity), given a known 1 kg mass and a spring scale?