Cell Biology Education
Vol. 3, 85–92, Summer 2004

Points of View

The Interface of Mathematics and Biology

Note from the Editors
Too often, biology has been considered by both students and faculty as the ideal major for the scientifically inclined but mathematically challenged, even though the advantage of quantitative approaches in biology has always been apparent. Increasingly, biologists are utilizing mathematical skills to create simulations or manage and query large data sets. The need for basic mathematical and computer science (CS) literacy among biologists has never been greater. But does this require a fundamental change in the organization of the undergraduate biology curriculum? What is the utility of math/CS in different areas of biology? How can we best provide math/CS instruction to biologists so that the utility is appreciated? Do all biology students require a stronger math/CS foundation, or only those interested in research careers? Given the speed at which technology changes, what is the best preparation? Three different points of view are offered below. Dr. Roger Brent, President and Director of the Molecular Sciences Institute, reflects on the "innumeracy" common among biologists and argues that significant insights into biological problems may be gained from better mathematical intuition. Professor Louis Gross, Department of Ecology and Evolutionary Biology and Department of Mathematics, University of Tennessee, has worked to engage all beginning biology students in quantitative analysis, to develop an appreciation of mathematical approaches. Professor Ronald Hoy, Department of Neurobiology and Behavior, Cornell University, has examined this problem from the perspective of a neurobiologist. We hope these different perspectives will stimulate discussion in biology departments around the world as to the best approach for our students.

Interdisciplinarity and the Undergraduate Biology Curriculum: Finding a Balance

Louis J. Gross
Departments of Ecology and Evolutionary Biology
and of Mathematics
The Institute for Environmental Modeling
University of Tennessee
Knoxville, TN 37996-1610
gross@tiem.utk.edu
http://www.tiem.utk.edu/~gross/

A variety of recent reports, notably *Bio2010* from the National Research Council (NRC, 2003; but see also *CUPM Curriculum Guide* [Mathematical Association of America, 2004], Salem and Dilts (2004), Meeting the Challenges (2003), and the collection of teaching material compiled by the Society for Mathematical Biology at http://www.smb.org/teaching/), recommend that undergraduate biology preparation become more interdisciplinary. This is based on the realization that modern biology research requires a breadth of skills that go well beyond the limited set of experiences that undergraduate biologists are exposed to in their traditional biology courses. This is not limited to biology, of course, for physical scientists have long been aware of the interconnections between fields and the reliance on quantitative approaches that go well beyond the basic undergraduate mathematics to which most physical science students are exposed.

In many respects, we have it worse in biology, though, because of the great diversity of topics from all areas of science that impinge on biology today. The days when bench scientists could claim that their students could lead fruitful research careers by learning appropriate skills and having good hands and lab technique are long gone. Similarly, despite the laments of E. O. Wilson about the lack of people being trained in taxonomy, it is next to impossible for those who are skilled field observers to make substantial new contributions to our understanding of natural system function without utilizing methods and approaches that natural historians of the past would never have dreamed of, let alone be skilled enough to use.

So if we take it as a given that modern biology research requires a diversity of perspectives and skills from areas of study outside the typical formal training that students receive in their life science courses, how should we proceed? The difficulty of this is exacerbated by the nationwide push to limit an undergraduate to 120 semester credit hours. How in the world do we squeeze into this limited time all that we think is necessary to prepare students to go on to successful graduate education or fruitful careers that don't require further formal education?

All experienced educators have heard, during discussions of curricular changes, the anxiety of colleagues about what has to be "left out." Central to this is the notion that students are empty vessels that information can be pumped into, that if this information isn't imbibed in your class they'll never be able to get it, and that they'll be hampered for life by this lack of exposure to the details of ribosome function or cell signaling. I am continually amazed that this naive view of students is prevalent, since virtually all educators argue as well that our goal should be developing students' capacity for critical thinking and problem-solving. If we were successful at that, surely students would then be able to ascertain both what knowledge they lack in order to investigate a particular problem and how to learn about the area (or collaborate with an expert in it).

Solutions, Solutions, at Least for Quantitative Training

Bio2010 is of two faces with regard to what might be called the "interdisciplinary gap"—the fact that students see the courses they take as discrete packages with few interconnections. On the one hand, the report recommends, as I have been arguing for years, that concepts from biology should be integrated within the quantitative courses that life science students take, and quantitative concepts should be emphasized throughout the life science curriculum. This provides what I have called "multiple routes to quantitative literacy," a process we have implemented in part at the University of Tennessee (for information see www.tiem.utk.edu/bioed/). One lesson from our experience is that it is possible to enhance a student's comprehension of quantitative topics by emphasizing these skills within the introductory general biology course. We have also redesigned the math sequence taken by many biology students to introduce a diversity of mathematical concepts, not just calculus, choosing the topics based on their utility in the broad life sciences.

The second face of the *Bio2010* report on this matter concerns the addition of numerous courses that students could take. As part of the math and CS recommendations, the report provides a list of specific concepts, offered presumably as a suggested template for topics to be included in the life science curriculum. On showing this list to mathematical colleagues, they all agree that they'd be overjoyed to have undergraduate math majors exposed to these and can't quite imagine how a life science undergrad would ever find the time to take all the courses covering these concepts. Indeed, none of the "proposed curricula" provided in *Bio2010* come close to including the variety of mathematical courses that would be necessary to cover these topics.

Happily, the math/CS section of *Bio2010* also includes a list of general quantitative principles useful to biology students, including rates of change, modeling, equilibria, stability, and stochasticity. My experience in developing our entry-level math for the life sciences course sequence is that it is indeed feasible (though the students certainly do have to work hard) to provide students with problems related to essentially all these general quantitative principles in a two-semester, three-credit hour sequence. Indeed, this sequence was developed by starting with a set of quantitative principles (not dissimilar to the list in *Bio2010*) developed in a workshop I organized in 1992 (see the report at http://www.tiem.utk.edu/~gross/Workshop92.recommendations.txt). This course sequence (see www.tiem.utk.edu/~gross/math151.html for the latest course materials) provides a rapid entrée to descriptive statistics, matrix algebra (including eigenvalues and eigenvectors), discrete modeling, and probability. It also provides the first experience our students generally have with the concept of an algorithm, through computer-based projects requiring the use of an appropriate mathematical software package (Matlab). Based on my experiences, integrating key quantitative concepts with life science examples within math courses, and repeating these concepts within life sciences courses in a variety of formats, is an approach that is feasible and fruitful.

So I urge us to specify the concepts, not the facts, and integrate these concepts throughout the curriculum in numerous formats. As just one example, when introducing maximization and minimization problems in our math sequence, I provide a quick summary of evolution by natural selection, mentioning Fisher's fundamental theorem. I don't expect the students to follow this in detail (they are first-year students who have had very few biology courses) but I do hope that in their later biology courses they will recall that evolutionary processes are somehow related to calculus and maximization problems. (Yes, I do mention constraints of history and urge them to read some of Gould's books too.)

Are Truly Interdisciplinary Courses Feasible?

As one alternative to the options discussed above, Bialek and Botstein (2004) argue for the development of truly interdisciplinary courses for biology students interested in a research career. They propose the development of courses that link math, the physical sciences, and biology as a substitute for separate disciplinary courses. Leaving aside for the moment the fact that few entering undergraduates have much of a clue what research is about, let alone whether they want to pursue it as a career, can we expect such interdisciplinary courses to be successful? Here the record is unfortunately quite discouraging. The National Science Foundation, as a follow-up to the Calculus-Reform initiative, in the mid-1990s supported the Mathematics Across the Curriculum program with large awards to several consortia. One goal of many of these consortia was to develop courses similar to the ones suggested by Bialek and Botstein (2004), though typically just integrating math and physics, math and chemistry, or some other combination. Although a few of these courses are still extant, there has been essentially no spread of them to other institutions, as there was with the calculus-reform courses. The potential reasons are numerous and not the focus of my comments here but include the "visionary burnout" that occurs when the persons most involved are no longer able to maintain the enthusiasm for interdisciplinarity that, to be successful, must have a strong, continual institutional commitment.

Frankly, I have a quite different view of undergraduates than do Bialek and Botstein (2004). It is a rarity for me to encounter an entering student who I can identify as a future researcher. Although this may be easy at certain institutions, I would argue strongly against focusing curricular reforms on the elite. I believe our objective in entry-level courses should be to entice students to see the connections between fields and to open their eyes to the facts that there are numerous open problems, that we (the entire scientific community) actually don't know that much about many areas of science, and that with hard work they can pursue a career in science that is tremendously satisfying and financially rewarding.

Getting Students into Research

In Sung and coworkers' (2003) discussion of the educational implications arising from interdisciplinary projects across the biological and physical sciences, several suggestions are provided to catalyze interactions between research groups. Their focus is strictly on graduate training and beyond, and it would be appropriate to supplement this with interdisciplinary projects for undergraduates.

What better mechanism could there be to aid bright young scientists to negotiate the barriers to interdisciplinary research than to offer opportunities for such research early in their education rather than waiting for graduate school? While such opportunities are enhanced through a variety of U.S. government-funded programs (the National Science Foundation's Research Experiences for Undergraduates initiative and a variety of minority education programs of the National Institutes of Health), these directly impact a very small fraction of the undergraduate pool. Suggestions for linking the science in programs such as these to educational research and details on a variety of case studies are given by in Avila (2003).

Classic undergraduate scientific training, providing a depth of experience in one disciplinary department (a "major"), feeds the associated desire of most graduate programs to obtain entering graduate students sufficiently versed in a field to be able to step directly into their graduate courses. There is a definite tension between this view of undergraduate education and the desire for breadth of experience, exposing students to the connections between fields in order to be able to appreciate (and carry out) modern science.

Bright students with sufficient gumption are now "speaking with their feet" and choosing double majors as the easiest means around the compartmentalization of knowledge that is the hallmark of much of undergraduate education. Multiple majors provide the quickest entrée to interdisciplinary fields. For over a decade, the students I find best prepared for graduate work in mathematical and computational ecology have backgrounds in both math and biology. Given the lethargic response of undergraduate curricula to the need for interdisciplinary training, motivated students are making the best of what the bureaucracy allows. Colleges that limit these options are living in the past, and many forward-thinking institutions are now growing double-degree and 5-year B.S.–M.S. programs specifically to meet the desires of their best students for breadth as well as depth.

While such double-degree programs are not the answer for all students, I have long argued (Gross, 1997) the utility of enhancing the interdisciplinary aspects of standard undergraduate disciplinary courses. A readily implemented method to enhance students' perceptions of the interconnections between fields and the unity of the scientific enterprise is the inclusion of biological case studies in mathematics courses and of more quantitative topics in biology courses (Gross, 2000). Even partial steps such as these should ease the transition to the types of interdisciplinary training that Sung *et al*. (2003) encourage at the graduate level and beyond. Still more effective are true undergraduate research experiences that cross disciplinary lines, not merely getting hands-on experience in a lab, but linking data collection, statistical analysis, and modeling. While few large institutions have the faculty resources to offer such experiences to the majority of biology majors, even a brief exposure to this as a case study within a quantitative or life science course is a positive step.

Some Not-So-Radical Proposals

Coming back to the issue of "fitting it all into 120 credits," the above suggestions provide at least a few mechanisms to assist educators. From a more global perspective, though, we have ourselves to blame. Over the past decades, there has been a strong push toward semester systems and away from quarter systems at U.S. institutions. The argument for this has often been to encourage "depth," and administrators also claim reduced costs through fewer registrations, though the evidence at my own institution does not indicate any significant financial savings arise. In the process we have sacrificed students options to branch out and investigate topics outside their major that they could not devote 15 weeks to but could spend 10 weeks on.

In developing future researchers, particularly in biology, the implication of *Bio2010* is that breadth of exposure to concepts from various fields should take precedence over depth. So it is past time to initiate a "back to quarters movement." At the least, discussion of such an option encourages our colleagues to acknowledge the importance of interdisciplinarity. To encourage this even further, we might urge our institutions to place tenure at the college or university level, rather than in a department, potentially easing the acceptance of colleagues who don't quite fit the mold of a single discipline yet are the best educators for a future generation of researchers.

REFERENCES

Avilia, B. K. B. (2003). *Integrating Research and Education: Biocomplexity Researchers Explore the Possibilities*, Washington, DC, National Academies Press. http://www.nap.edu/books/0309088712/html/.

Bialek, W., and Botstein, D. (2004). Introductory science and mathematics education for 21st-century biologists. Science *303*, 788–790.

Gross, L. (1997). Adopting an interdisciplinary paradigm: An interview. http://www.stolaf.edu/other/extend/Integration/gross.html.

Gross, L. J. (2000). Education for a biocomplex future. Science *288*, 807.

Mathematical Association of America (2004). Undergraduate Programs and Courses in the Mathematical Sciences: CUPM Curriculum Guide. http://www.maa.org/cupm/curr_guide.html.

Meeting the Challenges: *Education Across the Biological, Mathematical and Computer Sciences* (2003). http://www.maa.org/mtc/.

National Research Council (2003). *Bio2010: Transforming Undergraduate Education for Future Research Biologists*, Washington, DC, National Academies Press. http://www.nap.edu/books/0309085357/html/.

Salem, A., and Dilts, J. (2004). The Curriculum Foundations Workshop in Biology. http://www.maa.org/cupm/crafty/focus/cf_biology.html.

Sung, N. S., Gordon, J. I., Rose, G. D., Getzoff, E. D., Kron, S. J., Mumford, D., Onuchic, J. N., Scherer, N. F., Sumners, D. L., and Kopell, N. J. (2003). Educating future scientists. Science *301*, 1485.

DOI: 10.1187/cbe.04-03-0040

Intuition and Innumeracy

Roger Brent
The Molecular Sciences Institute
Center for Genomic Education and Computation
2168 Shattuck Avenue
Berkeley, CA 94704

The editors of *Cell Biology Education* requested a perspective on the math that biologists should know. As it happens, *Science* magazine just provided such perspective, superbly, when it published on Feburary 4 a number of thoughtful articles on math and biology. One, by Bialek and Botstein (2004), describes what would amount to comprehensive curriculum reforms to better integrate mathematics into the teaching of undergraduate biology and biology into the teaching of natural sciences and math. Another, by May (2004), covers some of the dangers practicing biologists face when attempting to use computational methods without a good understanding of the underlying mathematics. Here, I take those two articles as the starting context and raise only a few additional points. I argue that, even for biologists who lack mathematical skills, replacement of what is in some cases near–functional innumeracy with a better intuition about math and computers is possible and that doing so could have significant positive effects. I also make the case that significant gains in biological understanding may come from biologists taking onboard Twentieth-Century formalisms not now part of the canon of undergraduate mathematics.

Some Biologists Today Know Almost No Mathematics

In my opinion, the teaching reforms proposed by Bialek and Botstein (2004) would be exciting and effective and could be well implemented by the end of this decade at a number of elite universities. I fear, though, that the authors may be assuming a level of intellectual competency that may not exist for all students and teachers at all universities.

Here I am relying on my own experience. Not all biology graduate students come from undergraduate universities as good as the Princeton the authors write from, and some students from those universities who did enter graduate school did not perform well in undergraduate courses in math and physics. I'm under the impression that, at least during the 1990s, some of the graduate students at elite research universities self-identified in high school as not being good with mathematics and opted in university for a graduate school rather than a premedical track precisely to avoid the necessity to take and do well in those courses. Perhaps as a consequence, it is rather easy to ask evaluative questions that elicit disappointing answers. One, which I started asking in 1994, is to ask students how many molecules of their favorite proteins they are visualizing in cell nuclei or cytoplasms. Open mouths and expressions of shock and alarm when asked that question are not a good sign and provoke follow-up questions. Not infrequently, such follow-up questions establish the point that the student is at least a little bit uncomfortable with powers of 10, let us say, the difference among 100,000, 1 million, 10 million, 10^8, and 10^{11}. A friend has suggested that this behavior is not discomfort with powers of 10 so much as discomfort at the idea of being asked to make numerical estimates. Whatever its source, I now take such discomfort as operationally defining functional innumeracy in the lab.

In the Absence of Understanding, Intuition Can Sometimes Help

Happily, I believe there is a good deal of hope for providing some level of mathematical competence, even for scientists (such as the author) who lack talent or an aptitude for the subject.

Part of the reason for optimism is the boost provided by humankind's friend, the computer. Computer aids to performance of simulations and symbolic processing of equations (one could call these Matlab and Mathematica, respectively) can endow people with insight beyond their native math aptitude. These computational prostheses are not without their downsides. May (2004) raises a number of problems with them. One is that, for the naive, there may be embedded assumptions, such as "All distributions will conform to the central limit theorem," which are in many cases not true. Working with simulations based on these assumptions can lead to results that are clearly bogus, but some biologists, lacking good intuition, won't see the falsehood.

At the higher end of math talent, there may be other losses, or at least hazards, caused by reliance on simulation rather than analytical solution. May cites as an example the fact that most meteorologists believe that the outcomes of the Navier–Stokes equations used in weather models become chaotic after many weeks. The belief that the solutions are chaotic rests on the results of simulations rather than on formal proof. At Molecular Science Institute we have a running joke (except I am not sure it is funny) that if Karl Friedrich Gauss were alive today, he would have been so dependent on simulations that he never would have invented some of the math tools we use in our current work.

But at least at the moment, the relative lack of math skills in biology hardly constitutes a crisis. That is largely because much of the math now touted has limited utility to the problems biologists are actually interested in. The broad wavefront of molecular, cellular, and developmental biology is not suffering from the lack of a widely distributed ability to write down and solve differential equations to make quantitative models of the processes under investigation. The progress of scientists exhaustively testing combinations of growth factors and feeder layers to identify conditions that keep embyronic stem cells from differentiating (rightly) is not held back by the fact that they lack understanding of ideas arising from consideration of the emergent properties of spin glasses.

At this point, the worst problem may be that without skills or intuition, individual biologists may be vulnerable to snake oil and hype. A scientist giving a research seminar might try to convince more classically trained colleagues about the rightness of the use of a measure of mutual information such at the "receiver operator characteristic" to compare sets of genomic data. This statistic, which comes from early information theory, may not be sensible in cases where there is no information transmission, for example, for different series of mass spec peaks from different samples from different patients. A startup biotech company might claim that its pathway modeling software will provide a royal road that should lead within two years to the identification of new drug targets. Absent intution born of working with models, this assertion might be difficult to evaluate.

Better Intuition Would Carry Real Benefits

Intuition of the kind I describe has been created before. Perhaps the single best example is the change in U.S. science education following the launch of Sputnik. The relentless efforts to teach science and math throughout the school curriculum helped develop a generation of citizens with a broad scientific education. I have no idea how to document the assertion I am about to make, but it seems to me quite likely that the whole field of biology benefited from budding biologists whose post-Sputnik education had given them some kind of understanding of physics and math, machines, electronics, and computers. But 1957 was a long time ago.

Today, development of broader intuition among biologists would have positive consequences for government decisions in democratic polities. The relentless increase in biological capability has already ensured that the impact of biological research on human affairs can no longer be confined to ghettos labeled "health care," "biotechnology," or "bioethics." Over the next several decades, the dominant impacts of science on human affairs will come from developments in biology (Benford, 1995). Crafting political decisions that are not counterproductive (or even outright dangerous) will require engagement by biologists. Just as, in the 1950s and 1960s, the issues of the day (nuclear weapons, long-term effects of fallout, satellite reconnaisance) called for physicists, today the issues call for biologists. But unless those biologists are conversant at some level with other disciplines, including math, they won't be able to speak sensibly to the range of scientific knowledge that will be needed for good policy.

Because biology is so important, its needs are so great, and its researchers so numerous and so relatively well funded, it is also possible that the current drive to create better mathematical intuitions among biologists may have positive impacts outside of biology. A visionary architect of human computer interaction, Brenda Laurel, has called for interfaces that allow humans to interact better with complex models (2001). In her example, Laurel refers to representations of material and energy flows that might contribute to global climate change. However, should efforts to understand the quantitative behavior of biological systems bear fruit, biologists will have the same need to interact with the world of numbers represented inside computers. Consider the calculus that underlies any first-order mathematical formalization of a dynamic biological system. The teaching of rates of change has traditionally been founded in analytical geometry, parabolas, and ballistics, the thrown rocks so beloved to the male hominid. But perhaps there are other entries into the world of first derivatives and tipping points as different from the ballistic representation as the computer desktop metaphor is from the command line. To give an extreme example, in 10 years, an alternative computer-aided way in to interacting with the first and second derivative might combine muscle tension and proprioception: effort as one rides a bicycle up a hill, diminution of effort as the slope becomes shallower, decrease of effort to zero at the crown of the hill. Success in generating alternate metaphors might broaden the net cast by science education, for example, here by providing alternative means to teach precalculus to larger numbers of girls. The benefits to biology and to broader human activities from the development of methods and interfaces to heighten intuition would be sufficiently great as to justify the engagement of effort from the best designers, artists, and architects of the virtual world.

Not All the Desired Intuition May Come from Classical Math

It's also worth noting that gains might not come from math as it developed up to the nineteenth century, but from other formalisms developed by mathematicians and computer scientists during the last century to deal with the human-created world. Although the systematic analysis of these formalisms is relatively new (Simon, 1981), it is possible to ascribe to them certain common features; at least to me, they all seem messier than classical math, less Cartesian, precisely as if they have not had the benefit of the logical rigor that comes from centuries of theorem proving. At least three of these may hold promise: "control theory" (e.g., Doyle *et al*. 1992), "qualitative physics" (e.g., Kluwer and de Kleer [1990], whose Aristoliean picture of the world of physics intuitively seems a good match to natural language descriptions of molecular events by biologists), and the "qualitative calculus" developed by Kuipers (1994), which permits certain differential equation-like operations even on data that are incomplete (one knows that one quantity is bigger than another but not by how much or one knows the temporal sequence of events but not the exact times). Testing the merits of such formalisms is extremely demanding because it requires tight collaborations between biologists and the formalists, and in my opinion it remains an open question how useful these might be. But any approach that does have explanatory power may then benefit from being articulated in ways that biologists can use to make it intuitive and might merit inclusion in the undergraduate biology curriculums of the future.

Bigger payoffs might come from formalisms that have not yet been devised. Sussman (2001) has pointed out that mathematics has its origins in a workaday human activity, the geometry invented and practiced by Egyptian surveyors. Just as systematization of insights from this human activity led to algebra and the rigorous symbolic methods derived from it, so the Twentieth-Century development of computers and the imperative languages used to control them (Do this! Now do that! OK, now, if this happens, do this again!) will likely give rise to a whole new field of knowledge that arises from formal imperative languages. Certainly, there are cases in which procedural imperative languages have simplified education in electronics and mechanics (see Sussman, 2001).

In biology, the prize from such formalisms might be greater still. That is because the DNA tape in the genome is a set of commands in a procedural imperative language. Insights from a new discipline arising from twentieth-century computer programming might be deeply revealing. As biologists, we cannot lose sight of the ground truths of our field, the wondrous sets of particular facts about living systems that our ceaseless experimentation has revealed. But it may be true that only by educating ourselves about concepts from other disciplines can we create abstractions that help us rise above these particulars. If that is true, then it is math and computer science that must show us the way.

ACKNOWLEDGMENTS

I am grateful to Andrew Gordon, Larry Lok, and Gerry Sussman for thoughtful discussions. Work on the Alpha Project at MSI is supported by a Center of Excellence grant from the National Human

Genome Research Institute. Computer work receives support from the Defense Advanced Research Projects Agency Information Processing Technology Office.

REFERENCES

Benford, G. (1995). The biological century. Reason *November*. (Revised version: *Reason Online*, July 2001; http://reason.com/9511/BENFORDfeat.shtml.)

Bialek, W., and Botstein, D. (2004). Introductory science and mathematics education for 21st-century biologist. Science *303*, 788–790.

Doyle, F., Francis, B., and Tannenbaum, A. (1992). Feedback Control Theory, New York: Macmillan.

Kluwer, W. D., and de Kleer (1990). Readings in Qualitative Reasoning About Physical Systems, San Mateo, CA: Morgan Kaufmann Inc.

Kuipers, B. (1994). Qualitative Reasoning: Modeling and Simulation with Incomplete Knowledge, Cambridge, MA: MIT Press.

Laurel, B. (2001). Confessions of a Utopian Entrepreneur, Cambridge, MA: MIT Press.

May, R. M. (2004). Uses and abuses of mathematics in biology. Science *303*, 790–793.

Simon, H. R. (1981). The Sciences of the Artificial, 1st ed., Cambridge, MA: MIT Press.

Sussman, G. J. (2001). The legacy of computer science. Presented at NCR/CSTB Symposium on the Fundamentals of Computer Science; www7.nationalacademies.org/cstb/project_fcs_agenda3.html.

DOI: 10.1187/cbe.04-03-0041

New Math for Biology Is the Old New Math

Ron Hoy
Department of Neurobiology and Behavior
Cornell University
Ithaca, NY 14853

It's true. If you live long enough, you will see good ideas (and bad) come, go, and return again. So if you're old enough to remember the early 1960s songster/musical prankster, Tom Lehrer, you might remember his clever satire called "New Math," which lampooned a popular curriculum reform of the same name. New math's proponents championed the teaching of set theory, Boolean algebra, vectors, matrices and Markov chains, combinatorics, and a little splash of game theory. The motivating factor behind this revolutionary outlook was that these skills would come in handy in the physical sciences, social sciences, and—yes—life sciences. Fast-forward four decades to 2004 and, sure enough, such skills are even more useful now than in those post-Sputnik driven days of science education. But today, most of our students don't possess them—and most of them didn't back then, either. What happened? Did the new math movement wither away out of neglect or no respect? No, not completely. In fact, new math did get integrated in bits and pieces into undergraduate and even into high school curricula, surviving the scorn of "back to basics" movements. Various components of new math, in the form of "finite" or "discrete" mathematics courses, were "grafted" into undergraduate courses, especially those substituting for or supplementing the (still) traditional year of calculus, particularly in courses for non-science majors. So is there a problem here? Now, in the age of the "new biology," especially, systems biology, I think there is. Our biology students still tend to be math-averse and many of us instructors could stand a brushing-up, if not outright resuscitation, of our own math skills, which may have atrophied from disuse.

Part of the problem is that the old new math is still taught largely as a math or computer science course by math professors, and although examples from applied disciplines provide ample problem set fodder, students don't seem to transfer those skills into their biology courses as facilely as we might hope. Students need to relearn matrix operations anew, although admittedly, cognitive "savings" may make the (re)learning curve less daunting a climb. It might be useful to take a quick glimpse at curriculum reform in mathematics and see how it has fared since, say, 1960.

Dartmouth College has long been a "hot spot" for mathematics curriculum innovations. The Dartmouth mathematician John G. Kemeny was the lead author of a remarkable textbook, *Introduction to Finite Mathematics* (IFN), first printed in 1957. This book began with symbolic logic and truth tables, then introduced the reader to set theory. This was followed by combinatorial math ("partitions and counting") that led directly to probability theory. The final section took up vectors and matrices, which led to game theory and linear programming. The last chapter integrated all this nice math into applications in genetics (Markov chains), economics (game theory), and even some graph theory (communication networks). The prerequiste for IFN was 2.5 years of high school math (1950s vintage) because it was aimed at college freshmen. The book was revolutionary and it went through several editions before ending up in "remainder heaven" by the 1980s, out of print; it had become unfashionable, sadly. Nonetheless, finite math morphed into discrete math in subsequent decades and, with it, a heavier dose of graph theory. Calculus is not a prerequisite for discrete math and its techniques are highly relevant for today's life sciences. I would never deny the utility and necessity of calculus for natural science students, including biology, but discrete math can be learned independently of calculus and, I believe, may draw on cognitive skills on the continuum of mathematical literacy other than calculus—perhaps more user-friendly to the practical mind of a biologist. To finish my digression into history, John Kemeny later became president of Dartmouth College, but even then he continued to contribute to undergraduate math education by inventing the well-known computer language, BASIC, which was intended to be more a means to teach and learn programming than a programmer's working language, such as FORTRAN or C; BASIC is still around and useful in an object-oriented incarnation (e.g., Visual Basic). In the 1990s, Dartmouth mathematicians, led by Dorothy Wallace, proposed a visionary math-based curriculum reform called "Mathematics Across the Curriculum" (MATC; http://www.math.dartmouth.edu/~matc/). MATC was enthusiastically supported by the National Science Foundation's Department of Undergraduate Education.

So Dartmouth, in collaboration with other colleges and universities, formed a large consortium group and dedicated themselves to the goal of providing model curricula and supporting textbooks (from Key College Publishing). Thus, mathematics was integrated into courses in both physical and biological sciences, certainly, but also in humanities (English, art history, music, drama) and the social sciences. An impressive set of textbooks and curricula was developed and published (Wilson, 2000; National Research Council, 2003; Doyle, 2004). However, in spite of great dedication and enthusiasm by both students and faculty at Dartmouth, the part of MATC that most strongly engaged the physical science and engineering with math—the Integrated Math and Physics Program—was canceled by Dartmouth in 2002 (http://www.thedartmouth.com/article.php?aid=200204260103). Among the reasons given: lack of institutional funding to participating departments for additional faculty to provide for the added teaching load resulting from the creation of new, albeit innovative and even popular, courses. Perhaps there is a lesson here, albeit a sad one, for visionary curricular reform efforts. Clearly, the institution's administration must stand behind curriculum reform, a matter I'll return to at the end.

For the past 10 or 15 years, the teaching of college calculus has been the subject of a nationwide reform movement in mathematics departments across the country, which continues to the present. It has drawn many enthusiastic supporters and some detractors, for it remains controversial. Its aim is to teach the meaning of calculus concepts so that they can be confidently and successfully applied in the student's own discipline. Framing the concepts of calculus within real-world applications is a strong pedagogical theme. In some courses, in order to bring out the useful and powerful computational aspects of calculus as applied to the sciences and everyday life, instruction may include "crash courses" in useful computer software, such as Mathematica, Matlab, and Maple, and handheld calculators. The "cost" is to de-emphasize the rigorous but foundational aspects of formal proof in matters of continuity and differentiability, for example. This movement has, as part of its aim to increase the retention of students taking math postcalculus, to make math more attractive to students who might ordinarily be "turned off" or ignore math and to lower the number of students who drop out before completing the calculus sequence. The content of reform calculus is broadened to include examples from many disciplines, including biology. The reform goes beyond how the subject matter of calculus is reorganized; at many institutions, this is paired with new pedagogical practices, such as group learning in the context of small classes. Narrative-based materials such as the history of mathematical ideas and biographies of famous mathematicians supplement the traditional problems sets. Group learning is part of the "active learning" movement, itself a curriculum reform that, although several decades old, has begun to emerge as an important practice in the teaching of the physical and the life sciences.

What particular implications, if any, are there in these developments for addressing the mathematical training of twenty-first century life science undergraduates? Well, the new math, MATC, and calculus reform were centered primarily in mathematics departments. But is that sufficient? Perhaps it's time for biology faculty themselves to take some proactive steps and instruct their students in the necessary mathematics as situationally and pedagocially needed, rather than just leaving math instruction to the math department and hoping that their students will retain their math when they come to their biology courses. This might mean that that some biology faculty might need to shake the rust off their own math skills and learn some new ones. This does NOT mean that biologists should teach the math basics (we have enough to do as it is), but that they should reinforce what their math colleagues had taught their students earlier. How best to do this? Collaborative teaching between math and biology faculty is one way as I discuss later.

New Biology and the Old New Math

This could be an entire article by itself. I'll take just two examples.

Genomics. All areas of biology will feel the impact of genomic and proteomic science. The sheer volume of sequence data poses difficult and interesting challenges to biology researchers at all levels of biological analysis and all size scales, ranging from molecular to eco- and global systems. The wealth of sequences of DNA and RNA nucleotides and amino acids from cellular macromolecules and whole organism genomes and proteomes opens up undreamed of possibilities for comparative biology that boggle the mind. Making sense of sequence comparisons is the province of computational genomics and computational biology and is being implemented by wedding biology with the algorithmic expertise from mathematics, statistics, and computer science. The computations involve pretty fancy combinatorial mathematics and probability and the depiction of relatedness of sequences in the form of lineage and other phyletic trees. This ups the ante for knowledge of statistics and probablility needed by undergraduates. No longer is it sufficient to know the canonical descriptive statistics of normal distributions, such as measures of central tendency and variation, correlational measures, and hypothesis testing. The reams of molecular sequence data available on the Internet databases permit (or require) techniques such as factor and cluster analyses, principal components analysis, and Markov processes, to name a few. These techniques were presaged by educational visionaries like John Kemeny and his Dartmouth colleagues.

Neuroscience. The nervous system is the information superhighway of any animal more complex than a single cell. In humans, information flows as bioelectric currents that propagate throughout the living neural networks in the brain and spinal cord, which connect the body's sensory systems to its motor systems. The building blocks of our neural systems are single neurons that communicate with each other at anatomical and functional specializations called synapses. At synapses another form of information processing takes place that involves a network of chemical reactions occurring within each neuron partaking in communication—these "chemical cascades" are networks nested within each neuron (node) of an expansive interneuronal network that may includes hundreds or thousands of nodes, the totality and configuration of which vary on time scales ranging from micro- and milliseconds to seconds to minutes and longer. The chemical cascades within each neuron may lead to gene activation that may involve many genes, themselves forming a genetic network within the neuron's nucleus, which may ultimately lead to gene expression that leads to a cascade of protein synthesis (again, interacting within yet another network—of protein interactions in the cytoplasm of the neuron). The "end result" may be alterations in the structure and function of synapses, which manifest themselves as familiar behavioral changes such as learning and memory. If this seems baroque and complicated in terms of the different "players" and "pathways" of interaction, there is a branch of mathematics that can be applied to each step and size scale, with a generic set of computational tools to characterize each network. That is the province of *graph theory*.

Among systems biologists, it is thought that what is common to the various systems and levels of biology, at virtually any size scale, is the flow of information between and among many nodes. The instantiation or "snapshot" of a network at a particular place or time within the system can be called a "graph," not the familiar Cartesian version of orthogonal X and Y axes, but a network of "hubs" and interconnecting "roads." Whether the biological instantiation of a network under consideration is the pentose phosphate shunt in relation to glycolysis, or genetic networks and pleiotropic outcomes, or the control of salivation in relation to hearing a bell sound, or the effect of acid rain from the Midwest on the breeding cycle of trout in the Adirondack mountains, the underlying mathematics of graph theory is the same—that's what mathematics is supposed to be: generalizable and content—independent—which is it's power and beauty. But graph theory is also an abstract branch of mathematics. Isn't it already hard enough for biologists to master intracellular metabolic cycles, or the physiology of hippocampal neural systems involved in learning and memory, without climbing the difficult slopes of Mount Mathematics? The question is really whether some elements of graph theory can be taught, albeit without the formal rigor necessary for practicing mathematician, to biologists that can inform their comprehension of biological networks, or better, to enable them to articulate biological problems in a form that would lead to congenial collaborations with mathematicians who so understand graph theory deeply. It is certain that future research in life sciences will become ever more integrative and quantitatively rigorous, and hence necessarily collaborative, requiring the expertise of biologists, engineers, and computational scientists. At the very least, biologists should know enough mathematics to help formulate theory or to provide feedback by articulating biological constraints on the assumptions from which a mathematical model must begin. A good start would be for life scientists and mathematicians to develop synergistic teaching materials from the rich content of biology.

What to Do?

There are several ways to get this done, but all involve biology professors to refresh or learn anew some new (old) math. Increasingly, interdisciplinary courses are being created that are team-taught, acknowledging the obvious fact that it's getting harder for one person to teach even traditional courses in biology, such as developmental biology, cell biology, or neurobiology, and cover the multidisciplinary nature of modern life science research, which draws on physics, chemistry, computer science, engineering, and mathematics. Teaching teams need to include colleagues from these sister sciences. One of the pleasures of teaching as part of a team is that you receive a fresh look at familiar material—you learn something new, when you sit out there among the students, hearing your colleagues. This is very demanding of faculty time, however, and participation in interdisciplinary team teaching must be recognized as an unalloyed "good" by department chairs and deans. There needs to be a system of computational math workshops that target biology faculty. These could be organized either within one's home institution, if resources were sufficient, or by a consortium drawing from nearby institutions. These could be conducted during intersessions and summers. Again, local administration must acknowledge the payoff for modernizing life science courses (and its faculty) by encouraging and rewarding faculty that participate, by deferring costs to participate in workshops. Even more essentially, if a college is in full-fledged curriculum reform, its administration must support faculty-driven efforts, especially those faculty who risk stepping outside well-prescribed traditional disciplinary ways of scholarship and teaching. Anyone who has engaged in team teaching with faculty from other disciplines knows it takes much more time than teaching a solo course. Even though the number of actual lectures one prepares may be considerably less (e.g., half) than the number given in a solo course, one is obliged to go to every lecture and discussion section because not doing so vitiates any attempt at presenting a seamlessly integrated course. For anyone who's ever done this kind of teaching, it's exhilarating because it enables us to do what we do best—which is to broaden our horizons by learning new material and sharing our gains through teaching. But it takes time—administration should encourage participation in interdisciplinary courses by awarding full teaching credit even though the course "face time" may only be half of that in a solo course.

It is also worthwhile encouraging teaching teams who already have interdisciplinary expertise to develop instructional software in a modular format such that the modules could be the basis of "bootstrap" learning by both students and faculty. Such software should take advantage of the amazing graphical and animation capabilities in commercial development software to design didactic modules that engage dynamic visual approaches, as well as the computational and narrative approaches that are the ways of textbooks. We learn in all sorts of ways, and teaching software should exploit all avenues. The National Science Foundation has for almost two decades, championed multiple approaches toward the end of enhancing the math skills of life scientists, and innovative programs have resulted, including MATC at Dartmouth and its consortium colleges. There, the lead was taken by mathematicians and math departments. That's entirely appropriate and natural, but I'm saying that it's time that biologists and biology departments need to be proactive and work with their math colleagues, to upgrade their courses, computationally speaking. It will also require all biologists to get with the program, themselves.

REFERENCES

Doyle, P. (2004). The finite math collection. http://math.dartmouth.edu/~doyle/docs/carlyle/carlyle/node5.html.

National Research Council. (2003). *Bio2010: Transforming Undergraduate Education for Future Research Biologists*. Washington, DC: National Academies Press.

Wilson, R. (2000). The remaking of math. Chron High Educ. *46*(18).

DOI: 10.1187/cbe.04-03-0042