**STUDY AID I**

**Significant Figures**

Every measurement that we make has some inherent error due to the limitations of the measuring instrument and the experimenter. The numerical value recorded for a measurement should give some indication of the reliability (precision) of that measurement. In measuring a temperature using a thermometer calibrated at one-degree intervals we can easily read the thermometer to the nearest one degree, but we normally estimate and record the temperature to the nearest tenth of a degree (0.1°C). For example, a temperature falling between 23°C and 24°C might be estimated at 23.4°C. There is some uncertainty about the last digit, 4, but an estimate of it is better information than simply reporting 23°C or 24°C. If we read the thermometer as “exactly” twenty-three degrees, the temperature should be reported as 23.0°C, not 23°C, because 23.0°C indicates our estimate to the nearest 0.1°C. Thus in recording any measurement, we retain one uncertain digit. The digits retained in a physical measurement are said to be significant, and are called **significant figures**.

Some numbers are exact (have no uncertain digits) and therefore have an infinite number of significant figures. Exact numbers occur in simple counting operations, such as 5 bricks, and in defined relationships, such as 100 cm = 1 meter, 24 hours = 1 day, etc. Because of their infinite number of significant figures, exact numbers do not limit or determine the number of significant figures in a calculation.

**Counting Significant Figures.** Digits other than zero are always significant. Depending on their position in the number, zeros may or may not be significant. There are several possible situations:

1. All zeros between other digits in a number are significant; for example: 3.076, 4002, 790.2. Each of these numbers has four significant figures.

2. Zeros to the left of the first nonzero digit are used to locate the decimal point and are not significant. Thus 0.013 has only two significant figures (1 and 3); The zero is not significant.

3. Zeros to the right of the last nonzero digit, and to the right of the decimal point are significant, for they would not have been included except to express precision. For example, 3.070 has four significant figures; 0.070 has two significant figures.

4. Zeros to the right of the last nonzero digit, but to the left of the decimal, as in the numbers 100, 580, 37000, etc., may not be significant. For example, in 37000 the measurement might be good to the nearest 1000, 100, 10, or 1. There are two conventions which may be used to show the intended precision. If all the zeros are significant, then an expressed decimal may be added, as 580., or 37000. But a better system, and one which is applicable to the case when some but not all of the zeros are significant, is to express the number in exponential notation, including only the significant zeros. Thus for 300, if the zero following 3 is significant, we would write $3.0 \times 10^2$. For 17000, if two zeros are significant, we would write $1.700 \times 10^4$. The number we correctly expressed as 580. can also be expressed as $5.80 \times 10^2$. With exponential notation there is no doubt as to the number of significant figures.
**Addition or Subtraction.** The result of an addition or subtraction should contain no more digits to the right of the decimal point than are in that quantity which has the least number of digits to the right of the decimal point. Perform the operation indicated and then round off the number to the proper significant figure.

Example:  

\[
\begin{align*}
24.372 \\
+ 72.21 \\
+ 6.1488 \\
\hline
102.7308 \ (102.73)
\end{align*}
\]

Since the digit 1 in 72.21 is uncertain, the sum can have no digits beyond this point, so the sum should be rounded off to 102.73.

**Multiplication or Division.** In multiplication or division, the answer can have no more significant figures than the factor with the least number of significant figures. In multiplication or division, the position of the decimal point has nothing to do with the number of significant figures in the answer.

Example:  

\[
3.1416 \times 7.5 \times 252 = 5937.624 \ (5.9 \times 10^3)
\]

The operations of arithmetic supply all the digits shown, but this does not make the answer precise to seven significant figures. Most of these digits are not realistic because of the limited precision of the number 7.5. So the answer must be rounded to two significant figures, 5900 or \(5.9 \times 10^3\). It should be emphasized that in rounding-off the number you are not sacrificing precision, since the digits discarded are not really meaningful.

Example:  

\[
\frac{(27.52)(62.5)}{1.22} = 1409.836 \ (1.41 \times 10^3)
\]

The answer should contain three (3) significant figures.